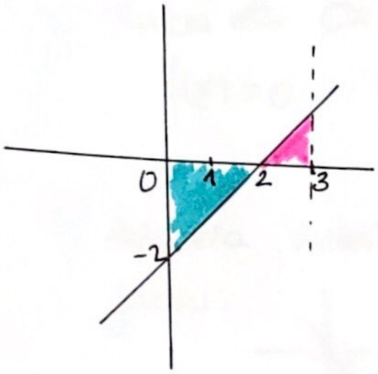
2. ADIBIDEA

$f(x) = x-2$ $[0,3]$ tartean eta OX ardatzak mugatutako azalera.



$*$ $$\int_0^2 (x-2)\,dx = \left[\frac{x^2}{2}-2x\right]_0^2 = \left(\frac{2^2}{2}-2\cdot 2\right) - \cancel{\left(\frac{0^2}{2}-2\cdot 0\right)} =$$

$$= \boxed{-2}$$ NEGATIBOA da OX ardatzaren azpian

$*$ $$\int_2^3 (x-2)\,dx = \left[\frac{x^2}{2}-2x\right]_2^3 = \left(\frac{3^2}{2}-2\cdot 3\right) - \left(\frac{2^2}{2}-2\cdot 2\right) =$$

$$= \boxed{\frac{1}{2}}$$ POSITIBOA da OX ardatzaren gainean.

$$A = \underbrace{-\int_0^2 (x-2)\,dx}_{} + \int_2^3 (x-2)\,dx = -(-2) + \frac{1}{2} = \boxed{\frac{5}{2}u^2}$$

azalera hau OX ardatzaren azpian dagoanez, ⊖ emoten dau.

3. ADIBIDEA.

$f(x) = 1-x^2$ funtzioak, OX ardatza eta $x=2$ eta $x=4$ zuzenek mugatutako azalera

$$\int_2^4 (1-x^2)\,dx = \left[x-\frac{x^3}{3}\right]_2^4 = \left(4-\frac{64}{3}\right) - \left(2-\frac{8}{3}\right) = 2-\frac{56}{3} = -\frac{50}{3}.$$

Azalera OX ardatzaren azpian dagoanez ⊖ emoten dau integrala berez honela adierazi daiteke

$$A = -\int_2^4 (1-x^2)\,dx = \left|\int_2^4 (1-x^2)\,dx\right| = \left|-\frac{50}{3}\right| = \boxed{\frac{50}{3}u^2}$$

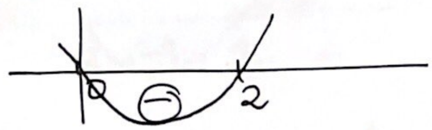
4 ADIBIDEA

1.) Kurba eta OX ardatzaren ebaki-puntuak

$$f(x)=0 \qquad x^2-2x=0$$

$$x(x-2)=0 < \begin{array}{l} x_1=0 \\ x_2=2 \end{array}$$

2.) Azalera ardatzaren goikaldean edo behekaldean dagoen jakin:



$$f(1)=1^2-2\cdot 1=-1$$

3.) Azalera kalkulatu

$$A=-\int_0^2 (x^2-2x)\,dx=\left[-\frac{x^3}{3}+x^2\right]_0^2=$$

$$=\left(-\frac{2^3}{3}+2^2\right)-\left(-\frac{0^3}{3}+0^2\right)=-\frac{8}{3}+4=\boxed{\frac{4}{3}\,u^2}$$

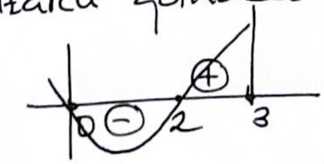
5. ADIBIDEA

1.) Kurba eta OX ardatzaren ebaki-puntuak

$$f(x)=0 \qquad x^2-2x=0$$

$$x(x-2)=0 < \begin{array}{l} x_1=0 \\ x_2=2 \end{array}$$

2.) Azalera ardatzaren goikaldean edo behekaldean dagoen jakin



$f(1)=-1$ −

$f(2{,}5)=1{,}25$ +

3.) Azalera:

$$A=-\int_0^2 (x^2-2x)\,dx+\int_2^3 (x^2-2x)\,dx=$$

$$=-\left[\frac{x^3}{3}-\frac{\cancel{2}x^2}{\cancel{2}}\right]_0^2+\left[\frac{x^3}{3}-\frac{2x^2}{2}\right]_2^3=-\left[\left(\frac{8}{3}-4\right)-(0-0)\right]+\left[(9-9)-\left(\frac{8}{3}-4\right)\right]=$$

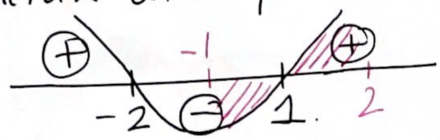
$$=-\frac{8}{3}+4-\frac{8}{3}+4=8-\frac{16}{3}=\boxed{\frac{8}{3}\,u^2}$$

6. ADIBIDEA

1.) Kurba eta OX ardatzaren ebakipuntuak

$$f(x)=0 \qquad x^2+x-2=0 \quad \begin{cases} x_1=-2 \\ x_2=1 \end{cases}$$

2.) Azalerak OX-en goinetik edo azpitik!



3.) Azalera:

$$A = -\int_{-1}^{1} (x^2+x-2)\,dx + \int_{1}^{2} (x^2+x-2)\,dx =$$

$$= -\left[\frac{x^3}{3}+\frac{x^2}{2}-2x\right]_{-1}^{1} + \left[\frac{x^3}{3}+\frac{x^2}{2}-2x\right]_{1}^{2} =$$

$$= -\left[\left(\frac{1}{3}+\frac{1}{2}-2\right)-\left(\frac{(-1)^3}{3}+\frac{(-1)^2}{2}-2(-1)\right)\right]+\left[\left(\frac{2^3}{3}+\frac{2^2}{2}-2\cdot 2\right)-\left(\frac{1^3}{3}+\frac{1^2}{2}+2\cdot 1\right)\right]$$

$$=\left(\cancel{-\frac{1}{3}}\cancel{-\frac{1}{2}}+2\cancel{+\frac{1}{3}}\cancel{+\frac{1}{2}}\cancel{+2}\right)+\left(\frac{8}{3}+\frac{4}{2}-4-\frac{1}{3}-\frac{1}{2}\cancel{-2}\right) = \boxed{\frac{31}{6}\,u^2}$$

=

7. ADIBIDEA

1.) $f(x)=0$ Kurba eta OX ardatzaren ebaki puntuak.

$$-x^2+4x-3=0 < \begin{matrix} x_1=1 \\ x_2=3 \end{matrix}$$

2.) OX-en gainetik edo azpitik!

3) Azalera.

$$A=\int_1^3 (-x^2+4x-3)\,dx = \left[-\frac{x^3}{3}+2x^2-3x\right]_1^3 =$$

$$=\left(-\frac{27}{3}+2\cdot 9-3\cdot 3\right)-\left(-\frac{1}{3}+2\cdot 1^2-3\cdot 1\right)=$$

$$=-\frac{27}{3}+9+\frac{1}{3}+1=-\frac{26}{3}+10=\boxed{\frac{4}{3}u^2}$$

8. ADIBIDEA

$f(x) = x^2 - 4$

$g(x) = -x^2 + 4.$

$f(x) = x^2 - 4$ | Erpina $x = \frac{-b}{2a} = 0$ $E(0,-4)$

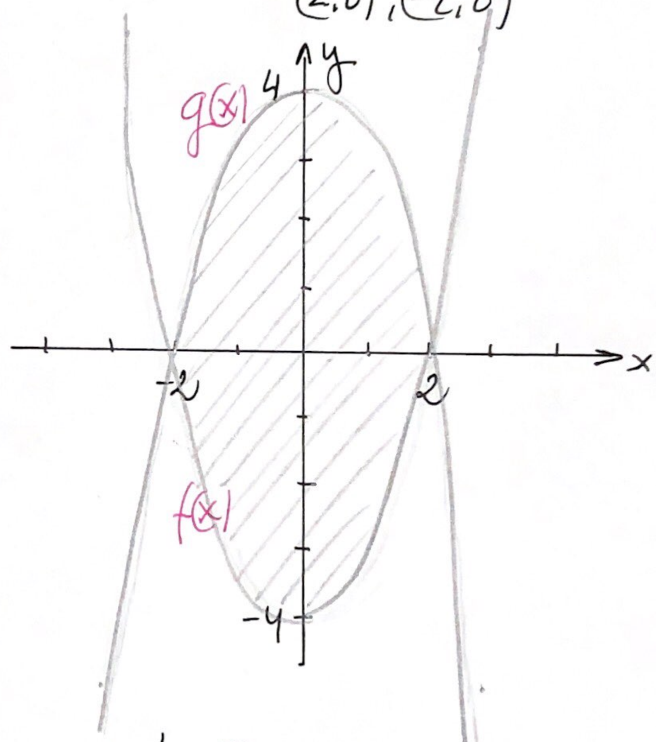
x ardat. ebaki puntuak $0 = x^2 - 4$ $x = \pm 2$

$(2,0), (-2,0)$

$g(x) = -x^2 + 4$ | Erpina $x = \frac{-b}{2a} = 0$ $E(0,4)$

x ard. ebaki puntuak $0 = -x^2 + 4$ $x = \pm 2$

$(2,0), (-2,0)$



1.) **f(x) eta g(x) ren ebaki puntuak**

$f(x) = g(x)$

$x^2 - 4 = -x^2 + 4$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$ $\boxed{\begin{array}{l} x_1 = 2 \quad y_1 = 0 \\ x_2 = -2 \quad y_2 = 0 \end{array}}$

2.) **Kenketa funtzioa**

goiko funtzioa − beheko funtzioa

$g(x) - f(x) = (-x^2 + 4) - (x^2 - 4) = -2x^2 + 8.$

3.) **Azalera - Barrow**

BARROW.

$$A = \int_{-2}^{2} (-2x^2 + 8)\,dx = \left[-\frac{2x^3}{3} + 8x\right]_{-2}^{2} = \left(-\frac{2 \cdot 2^3}{3} + 8 \cdot 2\right) - \left(-\frac{2 \cdot (-2)^3}{3} + 8(-2)\right)$$

$$= -\frac{16}{3} + 16 - \left(\frac{16}{3} - 16\right) = -\frac{16}{3} + 16 - \frac{16}{3} + 16 = \frac{64}{3}.$$

kontuz !!

9. ADIBIDEA $f(x) = y = x^2 \rightarrow$ Erpina $(0,0)$ Ahurra

$g(x) = y = 1$

1) Ebaki-puntuak $x^2 = 1 \quad x = \pm 1 < \begin{matrix}(1,1) \\ (-1,1)\end{matrix}$

2) Kenketa funtzioa

Esparrua funtzio biak bat datozen baliek mugatuko dabe

Goikoa - behekoa

$H(x) = 1 - x^2$

3) Azalera - Barrow

Azalera kalkulatzeko kontutan izan behar da zein funtzioak mugatzen dauan eremua goitik eta zeinek behetik.

Kasu honetan $g(x) = 1$ goitik, eta $f(x) = x^2$ behetik bestela azalera negatiboa izango litzateke.

$$A = \int_{-1}^{1} g(x) - f(x) = \int_{-1}^{1} (1 - x^2)\, dx = \left[x - \frac{x^3}{3} \right]_{-1}^{1} =$$

$$= \left(1 - \frac{1}{3}\right) - \left(-1 - \frac{(-1)^3}{3}\right) = 1 - \frac{1}{3} + 1 - \frac{1}{3} = \boxed{\frac{4}{3}\, u^2}$$

Ez bada kontutan izaten goikoa eta behekoa zein den, balio absolutuagatik izan behar da.

$$A = \left| \int_{-1}^{1} f(x) - g(x)\, dx \right| = \left| \int_{-1}^{1} (g(x) - f(x))\, dx \right|$$

Barrowen erregelagatik $A = \int_a^b f(x)\, dx = F(b) - F(a)$

10. ADIBIDEA

$f(x) = 1 - x^2$
$g(x) = 1 - x$

$f(x) = 1 - x^2$ Erpina $x = \frac{-b}{2a} = 0$ $E(0,1)$

$g(x) = 1 - x$ zuzena

x	0	1
y	1	0

1.) Ebaki puntuak

$f(x) = g(x)$ $\cancel{1} - x^2 = \cancel{1} - x$

$x^2 - x = 0$ $\begin{cases} x = 0 & (0,1) \\ x = 1 & (1,0) \end{cases}$

2.) Kenketa funtzioa goikoa - behekoa

$(1 - x^2) - (1 - x) = \cancel{1} - x^2 - \cancel{1} + x = -x^2 + x$

3.) Azalera

$A = \int_0^1 (1 - x^2) - (1 - x)\, dx = \int_0^1 -x^2 + x =$

Barrowen erregela aplikatuz

$A = \int_a^b f(x)\, dx = F(b) - F(a)$

$= \left[\frac{-x^3}{3} + \frac{x^2}{2} \right]_0^1 = \left(-\frac{1^3}{3} + \frac{1^2}{2} \right) - \cancel{\left(\frac{-0^3}{3} + \frac{0^2}{2} \right)}^{0} = \boxed{\frac{1}{6} u^2}$

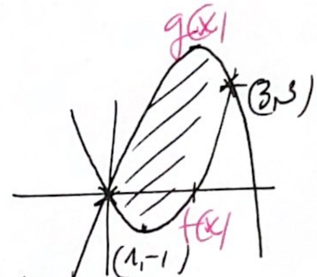
$\boxed{\text{11. ADIBIDEA}}$

$$f(x) = x^2-2x$$
$$g(x) = -x^2+4x.$$

1. Ebakitza puntuak

$$f(x) = g(x)$$
$$x^2-2x = -x^2+4x \nearrow \begin{array}{l} 2x^2-6x=0 \\ 2x(x-3)=0 \end{array} \left\langle \boxed{\begin{array}{ll} x_1=0 & (0,0) \\ x_2=3 & (3,3) \end{array}} \right.$$

~~2~~. Kurben irudikapena egiteko:



$f(x) = x^2-2x$ $\cup$ | Erpina $x = \frac{2}{2} = 1$ $E(1,-1)$

x ard. ebakitza punt:

$$0 = x^2-2x \left\langle \begin{array}{l} x_1=0 \\ x_2=2 \end{array} \right.$$

$g(x) = -x^2+4x$ $\cap$ | Erpina $x = \frac{-4}{2(-1)} = 2$ $E(2,4)$

x ardatzapot:

$$0 = -x^2+4x \left\langle \begin{array}{l} x_1=0 \\ x_2=4 \end{array} \right.$$

2. Kenketa puntuak

$$g(x)-f(x) = (-x^2+4x)-(x^2-2x) = -2x^2+6x$$

3. Atalno: Barroren erregela

$$A = \int_0^3 (g(x)-f(x))\,dx = \int_0^3 (-x^2+4x)-(x^2-2x)\,dx =$$

$$= \int_0^3 (-2x^2+6x)\,dx \overset{\text{BARROW}}{=} \left[\frac{-2x^3}{3} + \frac{6x^2}{2}\right]_0^3 =$$

$$= \left(-\frac{2\cdot 3^3}{3} + 3\cdot 3^2\right) - \overset{0}{\cancel{\left(\frac{-2\cdot 0^3}{3} + \frac{6\cdot 0^2}{2}\right)}} = \boxed{9u^2}$$